Self–Fields and Their Effects on Electron Orbits in a Three-Dimensional Helical Wiggler Free-Electron Laser

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***Abstract*—A theory for self–fields in a free–electron laser with three-dimensional (realistic) helical wiggler and axial magnetic field is presented. It is shown that the transverse–velocity–induced self–magnetic field has a diamagnetic effect for group I orbits, while for group II orbits it has a paramagnetic effect.**

I. Introduction And Background

The free-electron laser (FEL) is a device in which a relativistic electron beam radiates as a result of oscillations induced by its passage through a wiggler magnetic field [1]. High-density relativistic electron beams employed in FELs produce strong collective effects associated with the self–electric and self–magnetic fields of the beams. Self-fields can induce chaos in a helical wiggler FEL [1]. Recently, some theoretical studies of self–fields in a helical wiggler with axial magnetic field or ion–channel guiding have been carried out [2-4]. All of the above cited studies are based on a helical wiggler in an idealized (one-dimensional) approximation.

The purpose of the present paper is to study the self–fields in a FEL with three-dimensional helical wiggler and axial magnetic field.

II. Self-fields

The self–electric field induced by the steady–state charge density of the non–neutral electron beam can be obtained by solving Poisson's equation

$$\nabla \cdot \mathbf{E}_s = -4\pi e\, n_b(r), \tag{1}$$

where –e is the electron charge and n_b is the electron number density. Solving Eq. (1) for an electron beam with uniform density profile, we obtain

$$\mathbf{E}_s = -2\pi e n_b (x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y). \tag{2}$$

The self–magnetic field generated by the steady–state current density of the beam may be determined by Ampere's law

$$\nabla \times \mathbf{B}_s = \frac{4\pi}{c}\mathbf{J} = -4\pi e n_b \boldsymbol{\beta}, \tag{3}$$

where c is the speed of light in vacuum and $\boldsymbol{\beta} = \mathbf{v}/c$ is the normalized electron velocity. The lowest–order representation for the self–magnetic field is determined under the assumption that the electron beam propagates with constant velocity $\mathbf{v} = v_\parallel \hat{\mathbf{e}}_z$. In this case, the solution of Eq. (3) may be expressed in the form

$$\mathbf{B}_s \equiv \mathbf{B}_{s\parallel} = 2\pi e n_b \beta_\parallel (y\hat{\mathbf{e}}_x - x\hat{\mathbf{e}}_y), \tag{4}$$

where $\beta_\parallel = v_\parallel / c$ is the normalized electron axial velocity. The above equation represents the self–magnetic field generated by the axial velocity. The relativistic motion of an electron in a FEL with helical wiggler and axial magnetic filed in the presence of self–electric field and self–magnetic filed induced by electron axial velocity will be based on equation of motion

$$\frac{d(m\gamma\mathbf{v})}{dt} = -e\left[\mathbf{E}_s + \frac{1}{c}\mathbf{v}\times(\mathbf{B}_w + B_0\hat{\mathbf{e}}_z + \mathbf{B}_{s\parallel})\right]. \tag{5}$$

Here γ is the Lorentz relativistic factor,

$$\mathbf{B_w} = 2B_w\Big[I'(k_w r)\cos\chi\hat{\mathbf{e}}_\mathbf{r} - I_1(k_w r)\left(\frac{\sin\chi}{k_w r}\hat{\mathbf{e}}_\theta - \sin\chi\hat{\mathbf{e}}_\mathbf{z}\right)\Big] \tag{6}$$

is the three-dimensional helical wiggler magnetic field in cylindrical coordinates, where $\mathbf{B}_w$ is the amplitude of the wiggler field, $\chi \equiv \theta - k_w z$, $\lambda_w (\equiv 2\pi / k_w)$ is the wiggler wave length and I_1 , I'_1 are the modified Bessel function of the first kind (of order 1) and its derivative, respectively.

The steady–state solution of Eq. (5) for the normalized electron velocity can be obtained as

$$\boldsymbol{\beta}^{(0)} = \beta_w^{(0)}(\hat{e}_r\cos\chi + \hat{e}_\theta\sin\chi) + \beta_z^{(0)}\hat{e}_z \tag{7}$$

where $\beta_z^{(0)} \equiv \beta_\parallel = const.$ and

$$\beta_w^{(0)} \equiv \frac{2\Omega_w\beta_\parallel^2 I_1(\lambda_0)/\lambda_0}{\Omega_0\beta_\parallel - \omega_b^2(1-\beta_\parallel^2) - \beta_\parallel^2 \pm 2\Omega_w\beta_\parallel I_1(\lambda_0)} \tag{8}$$

is the magnitude of the normalized electron transverse velocity induced by the wiggler magnetic field in the presence of the self–electric field and axial velocity–induced self–magnetic filed. In Eq. (8) $\Omega_w \equiv eB_w / m\gamma k_w c^2$ is the normalized wiggler magnetic field, $\Omega_0 \equiv eB_0 / m\gamma k_w c^2$ is the normalized axial magnetic filed, $\lambda_0 \equiv \mp\beta_w^{(0)}/\beta_\parallel$ and $\omega_b \equiv (2\pi e^2 n_b / m\gamma k_w^2 c^2)^{1/2}$ is the normalized electron beam frequency.

The transverse electron velocity induced by the wiggler magnetic field generates another self–magnetic filed known as wiggler–induced self–magnetic field. Substituting Eq. (7) into Ampere's law [Eq. (3)] and using self-consistent method and after some algebra the wiggler–induced or transverse-velocity induced self-magnetic field can be obtained as

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$$\mathbf{B}_{s\perp} \equiv \frac{4\omega_b^2 \beta_\parallel^2 I_1(\lambda)/\lambda}{\Omega_0\beta_\parallel - \beta_\parallel^2 - \omega_b^2(1+\beta_\parallel^2) \pm 2\Omega_w \beta_\parallel I_1(\lambda)} B_w \hat{\mathbf{e}}_\theta, \qquad (9)$$

where $\lambda \equiv \mp\beta_w/\beta_\parallel$ and β_w is the magnitude of transverse electron velocity in the presence of total self-fields ($\mathbf{E}_s + \mathbf{B}_{s\parallel} + \mathbf{B}_{s\perp}$). The effects of total self-field on electron velocity can be obtained by solving Lorentz equation using Eqs. (2), (4) and (9). This yields

$$\boldsymbol{\beta} = \beta_w(\hat{e}_r \cos\chi + \hat{e}_\theta \sin\chi) + \beta_\parallel \hat{e}_z, \qquad (10)$$

where

$$\beta_w \equiv \frac{2\Omega_w \beta_\parallel^2 I_1(\lambda_0)/\lambda_0}{\Omega_0\beta_\parallel - \omega_b^2(1+\beta_\parallel^2) - \beta_\parallel^2 \pm 2\Omega_w\beta_\parallel I_1(\lambda)}. \qquad (11)$$

Equation (11) shows resonant enhancement in the magnitude of the transverse velocity when

$$\Omega_0\beta_\parallel - \omega_b^2(1+\beta_\parallel^2) - \beta_\parallel^2 \pm 2\Omega_w\beta_\parallel I_1(\lambda) = 0 \qquad (12)$$

which separates the electron orbits into two groups, namely group I and group II, that are defined by

$$\Omega_0 < [\omega_b^2(1+\beta_\parallel^2) + \beta_\parallel^2 \mp 2\Omega_w\beta_\parallel I_1(\lambda)], \qquad (13)$$

and

$$\Omega_0 > [\omega_b^2(1+\beta_\parallel^2) + \beta_\parallel^2 \mp 2\Omega_w\beta_\parallel I_1(\lambda)], \qquad (14)$$

respectively.

III. Results and Discussion

Figure 1 shows the graph of the normalized transverse–velocity–induced self–magnetic field $\overline{\mathrm{B}}_{s\perp} (\equiv \mathrm{B}_{s\perp}/\mathrm{B}_w)$ as a function of the normalized axial magnetic field Ω_0 for parameters shown in the figure. As shown in Fig. 1 the normalized transverse–velocity–induced self–magnetic field is negative for group I orbits and is positive for group II orbits. Therefore, the transverse–velocity–induced self–magnetic field decreases the effective wiggler magnetic field and has a diamagnetic effect for group I orbits, while for group II orbits it increases the effective wiggler magnetic field and has a paramagnetic effect. Figure 2 shows the normalized axial velocity $\beta_\parallel$ in the presence and absence of self-fields. The normalized axial velocity $\beta_\parallel$ for may be obtained from the conservation of energy ($\gamma^{-2} = 1 - \beta_w^2 - \beta_\parallel^2$) using Eq. (11). As Fig. 2 shows, self-fields can significantly affect the electron orbits.

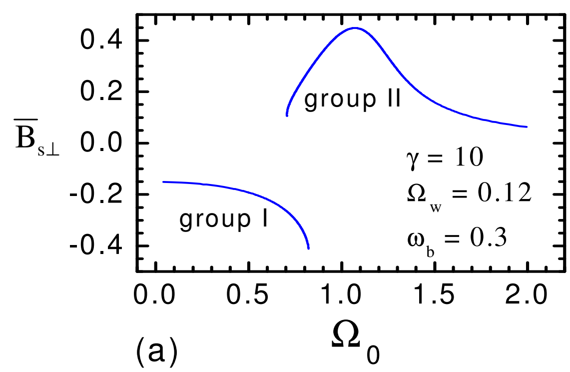

Fig. 1 Graph of the normalized transverse-velocity-induced self-magnetic field $\overline{\mathrm{B}}_{s\perp}$ as a function of the normalized axial magnetic field Ω_0 for parameters shown in the figure.

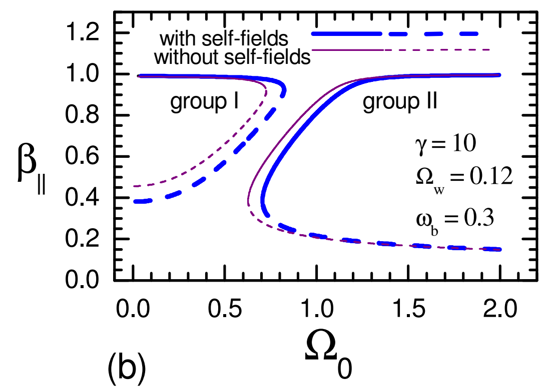

Fig. 2 Graphs of the normalized electron axial velocity $\beta_\parallel$ in the presence and absence of self-fields as a function of the normalized axial magnetic field Ω_0. The dashed lines indicate unstable orbits.

References

[1] H. P. Freund and J. M. Antonsen, Jr., *Principles of free electron lasers* (Chapman and Hall, London, 1996).

[2] M. Esmaeilzadeh, H. Mehdian ,J. E. Willett, and Y. M. Aktas, Phys. Plasmas, **10**, 905 (2003).

[3] M. Esmaeilzadeh, J. E. Willett, and L. J. Willett, J. Plasma Phys. **72**, 59 (2006).

[4] S. Mirzanejhad, B. Maraghechi, and T. Mohsenpour, Phys. Plasmas **11,** 4777 (2004).

[5] M. Esmaeilzadeh, S. Ebrahimi, A. Saiahian ,J. E. Willett, and L.J. Willett, Phys. Plasmas, **12**, 093103 (2005).